Hadron Resonance Gas Model, Thermodynamics of QCD and Heavy Quark Physics

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Excited QCD 2016

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Issues

1. QCD Thermodynamics and Hadron Spectrum
   - Motivation
   - Hadron Spectrum
   - Symmetries in QCD: the Polyakov loop
2. Hadron Resonance Gas Model
   - Quantization of multiquark states
   - Hadron Resonance Gas Model for the Polyakov loop
   - Higher representations: Casimir scaling
3. Heavy $\bar{Q}Q$ free energy: hadronic representation
   - Heavy $\bar{Q}Q$ interaction
   - Heavy $\bar{Q}Q$ free energy
   - Avoided crossing

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References

- **QCD at finite T:** Phys.Lett. B563 (2003) 173-178, Phys.Rev. D69 (2004) 116003,
- **Polyakov-Nambu–Jona-Lasinio:** hep-ph/0410053; AIP Conf.Proc. 756 (2005) 436-438, Phys.Rev. D74 (2006) 065005, Rom.Rep.Phys. 58 (2006) 081-086. PoS JHW2005 (2006) 025. AIP Conf.Proc. 892 (2007) 444-447. Eur.Phys.J. A31 (2007).
- **Dim-2 Condensates:** JHEP 0601 (2006) 073, Phys.Rev. D75 (2007) 105019. Indian J.Phys. 85 (2011) 1191-1196. Nucl.Phys.Proc.Suppl. 186 (2009) 256-259. Phys.Rev. D81 (2010) 096009.
- **Hadron Resonance Gas for Polyakov loop:** Phys.Rev.Lett. 109 (2012) 151601. AIP Conf.Proc. 1520 (2013) 185-190. Nucl. Phys. Proc. Suppl. 234 (2013) 313-316. Acta Phys. Polon. B 45, 2407 (2014)
- **Polyakov loop Spectroscopy:** Phys.Rev. D89 (2014) 076006, Nucl. Part. Phys. Proc. 258-259 (2015). 109.
- **Heavy Quark Physics:** arXiv:1507.08606[hep-ph].

Issues

Motivation

- The partition function of QCD

$$Z_{\mathrm{QCD}} = \mathrm{Tr}\, e^{-H_{\mathrm{QCD}}/T} = \sum_n e^{-E_n/T} \,, \qquad H_{\mathrm{QCD}}\psi_n = E_n\psi_n \,,$$

- Spectrum of QCD ⟶ Thermodynamics

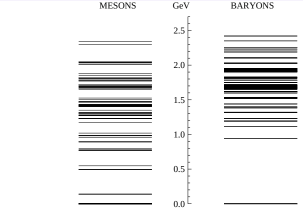
  Hadron Resonance Gas Model

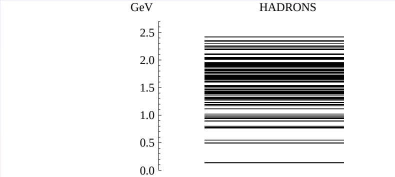
- In the confined phase: Colour singlet states (hadrons + $\cdots$???)
- In the deconfined phase: quarks and gluons ⟶ quark-gluon plasma.
- Phase transition is a crossover ⟶ Do we see quark-gluon substructure BELOW the "*phase transition*"?

Issues

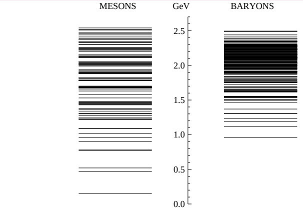
Hadron Spectrum (u,d,s)

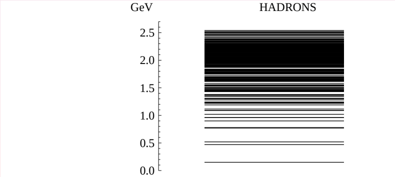
- Particle Data Group (PDG) compilation





- Relativized Quark Model (RQM), Isgur'85





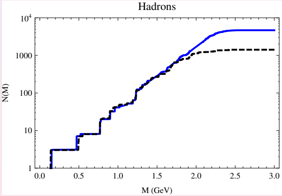
Cumulative number of states

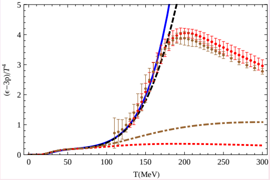
- Cumulative number $\equiv$ number of bound states below M.

$$N(M) = \sum_n \Theta(M - M_n) \,,$$

- Which states count?

$$N(M) = N_{\bar{q}q}(M) + N_{qqq}(M) + \cdots \,,$$





$$N_{\bar{q}q} \sim M^6 \,, \qquad N_{qqq} \sim M^{12} \,, \qquad N_{\bar{q}q\bar{q}q} \sim M^{18} \quad \text{and} \quad N_{\text{hadrons}} \sim e^{M/T_H}$$

$$T_H \sim 150\,\text{MeV} \equiv \text{Hagedorn temperature}$$

- Non-interacting Hadron-Resonance Gas works for $T \lesssim 0.8 T_c$.

Issues

Symmetries in QCD

The chiral-deconfinement cross over is a unique prediction of lattice QCD

- Order parameter of chiral symmetry breaking ($m_q = 0$)
  Quark condensate $SU_L(N_f) \otimes SU_R(N_f) \to SU_V(N_f)$

$$\langle \bar{q}q \rangle \neq 0 \quad T < T_\chi \,, \qquad \langle \bar{q}q \rangle = 0 \quad T_\chi < T \,.$$

- Order parameter of deconfinement ($m_q = \infty$)
  Polyakov loop: Center symmetry $\mathbb{Z}(N_c)$ broken

$$L_3 = \frac{1}{N_c} \langle \mathrm{tr}_c e^{iA_0/T} \rangle = 0 \quad T < T_D \,, \qquad L_3 = \frac{1}{N_c} \langle \mathrm{tr}_c e^{iA_0/T} \rangle \neq 0 \quad T_D < T \,,$$

Phase transition in QCD: $T_\chi \approx T_D$ at least when $\mu = 0$.

- $T < T_c$: confined phase

$$\nexists \; \chi \text{ Sym } (\langle \bar{q}q \rangle \neq 0)$$

$$\exists \; \mathbb{Z}(3) \text{ Sym } (L_3 = 0)$$

- $T_c < T$: unconfined phase

$$\exists \; \chi \text{ Sym } (\langle \bar{q}q \rangle = 0)$$

$$\nexists \; \mathbb{Z}(3) \text{ Sym } (L_3 \neq 0)$$

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Chiral Quark Models at Finite Temperature

- Chiral Perturbation Theory $\longrightarrow$ Suppose the non-vanishing of chiral condensate. It cannot describe the QCD phase transition.
- Chiral Quark Models $\longrightarrow$ Dynamics of QCD at low energies (low temperatures).
- Mean field approximation. Minimal coupling of Polyakov loop (analogy with chemical potential). Ogilvie and Meissinger PLB (1995) K. Fukushima, PLB591, 277 (2004). W. Weise et al. PRD73, 014019 (2006), N. Scoccola, D. G. Dumm (2008), S.K. Ghosh et al. PRD73, 114007 (2006),
- Quantum and local Polyakov loop. E.Megías, E.Ruiz Arriola and L.L.Salcedo, PRD74: 065005 and 114014 (2006).

Polyakov-Constituent Quark and Gluon Model

[Fukushima '04], [Megías et al. '06], [Ratti et al. '06], [Sasaki et al. '12]

- Gluodynamics + Polyakov-Constituent Quark Model:

$$Z = \int \mathcal{D}\Omega \mathcal{D}q \, e^{-S(T,\Omega)} , \qquad S(T,\Omega) = S_G(T,\Omega) + S_q(T,\Omega) .$$

with

$$S_G(T,\Omega) = -2T \int \frac{d^3x d^3p}{(2\pi)^3} \operatorname{tr} \log \left(1 - \Omega_8(\vec{x}) \, e^{-E_p/T} \right) ,$$

$$S_q(T,\Omega) = 2N_f \int \frac{d^3x d^3p}{(2\pi)^3} \left(\operatorname{tr} \log \left[1 + \Omega_3(\vec{x}) \, e^{-E_p/T} \right] + \operatorname{tr} \log \left[1 + \Omega_3^\dagger(\vec{x}) \, e^{-E_p/T} \right] \right) .$$

- Assume correlation domains of size V/T **[Megías et al. PRD74 '06]**:

$$Z = \int \prod_n d\Omega_n \, e^{-\sum_n S_n} , \qquad S_n = (V/T)\mathcal{L}(x_n) ,$$

$$\langle S_n S_{n'} \rangle = \begin{cases} \langle S_n^2 \rangle & n = n' \\ \langle S_n \rangle \langle S_{n'} \rangle & n \neq n' \end{cases} .$$

Expansion in the number of constituents

Polyakov loop $\Omega(\mathbf{x}) = e^{-\beta A_0(\mathbf{x})} \equiv$ quantum and local degree of freedom.

- Multi-quark/gluon states: Create/Annihilate a quark at point $\vec{x}$ and momentum $\vec{p}$ (also gluons)

$$\Omega_{3(8)}(x) e^{-E_P/T} \,, \qquad \Omega_3(x)^+ e^{-E_P/T} \,.$$

After a series expansion:

$$S_c(\Omega, T) = -g_c \zeta_c V \sum_{n=1}^{\infty} \frac{(\lambda \zeta_c)^n}{n} J_n(M_c, T) \mathrm{tr}(\Omega_c^n(\mathbf{x})) \quad \text{with} \quad J_n(M_c, T) \sim e^{-nM_c/T}$$

with $c = q, \bar{q}, G$. $\zeta_c = \pm 1$ for bosons(fermions).
This is an expansion in the number of constituents.

- Boltzmann factors ⟶ Multi-quark and multi-gluon states:
  - Meson contributions: $Z_{[\bar{q}q]} \sim \langle \mathrm{tr}\, \Omega_3 \; \mathrm{tr}\, \Omega_3^\dagger \rangle \sim e^{-(M_q + M_{\bar{q}})/T}$.
  - Baryon contributions: $Z_{[qqq]} \sim \langle \mathrm{tr}\, \Omega_3^{N_c} \rangle \sim e^{-N_c M_q/T}$.
  - Glueball contributions: $Z_{[GG]} \sim \langle \mathrm{tr}\, \Omega_8 \; \mathrm{tr}\, \Omega_8 \rangle \sim e^{-2M_G/T}$.
  - ... Exotic states: $Z_{[\bar{q}q\bar{q}q]} \equiv$ tetraquarks, $Z_{[q^3 q\bar{q}]} \equiv$ pentaquarks, $\cdots$

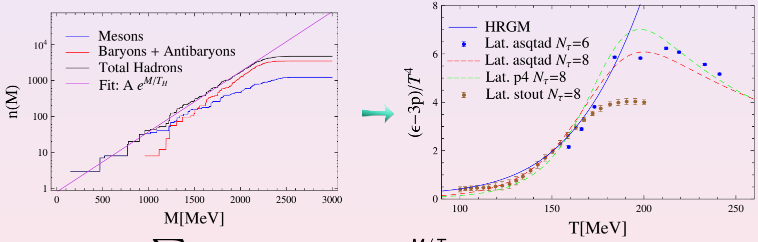
Hadron spectrum and Trace Anomaly

- Contributions $[\bar{q}q]$, $[qqq]$, ... to the partition function:

$$Z \simeq Z_{[\bar{q}q]} Z_{[qqq]} \cdots$$

$\Longrightarrow$ Gas of Non Interacting Hadrons $\Longrightarrow$ Hadron Resonance Gas Model.

Hadron spectrum with light u, d, s quarks, and Trace Anomaly.



$$n(M) = \sum_\alpha g_\alpha \Theta(M - M_\alpha) \simeq A\, e^{M/T_H} \, ,$$

$$\frac{\epsilon - 3p}{T^4} = \sum_{k=1}^{\infty} \int dM \left(\frac{\partial n_m(M)}{\partial M} + (-1)^{k+1} \frac{\partial n_b(M)}{\partial M} \right) \frac{1}{2k\pi^2} \left(\frac{M}{T} \right)^3 K_1 \left(k \frac{M}{T} \right) .$$

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Polyakov loop and the HRG model, PRL 109 (2012)

- For the Polyakov loop in the fundamental representation

$$L_3(T) = \left\langle \frac{1}{N_c} \mathrm{tr}\, \Omega_3 \right\rangle = 2N_f \int \frac{d^3x d^3p}{(2\pi)^3} e^{-E_p/T} \frac{1}{N_c} \underbrace{\langle \mathrm{tr}\, \Omega(\vec{x}_0)\, \mathrm{tr}\, \Omega^{-1}(\vec{x}) \rangle}_{e^{-\sigma|\vec{x}_0 - \vec{x}|/T}} + \dots$$

$$= \frac{2N_f}{N_c} \int \frac{d^3x d^3p}{(2\pi)^3} e^{-H(\vec{x},\vec{p})/T} + \dots$$

- A single q Hamiltonian

$$H(\vec{x},\vec{p}) = \sqrt{\vec{p}^2 + M^2} + V_q(r) \overset{\text{Quantization}}{\Longrightarrow} H\psi_\alpha = \Delta_\alpha \psi_\alpha$$

- Hadron Resonance Gas Model for the Polyakov loop:

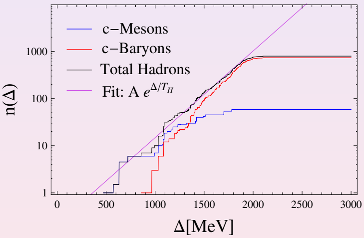
$$L_3(T) \simeq \frac{1}{2N_c} \sum_{h,\alpha} g_{[h\alpha]} e^{-\Delta_{[h\alpha]}/T}, \qquad \Delta_{[h\alpha]} = \lim_{m_h \to \infty} (M_{[h\alpha]} - m_h)\,.$$

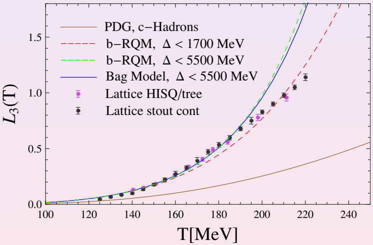
$|h,\alpha\rangle \equiv$ Heavy-light system $\longrightarrow$ Spectrum of mesons and baryons with 1 heavy quark "h" + dynamical quarks "α".

Polyakov loop and the HRG model, PRL 109 (2012)

$$[h\alpha] = \underbrace{[h\bar{q}]}_{\text{Mesons}}, \underbrace{[hqq]}_{\text{Baryons}}, \underbrace{[h\bar{q}q\bar{q}]}_{\text{Tetraquarks}}, \cdots$$

Hadron spectrum with 1 heavy quark and several light quarks (RQM Isgur model), and Polyakov loop.





$$n(\Delta) = \sum_\alpha g_{[h\alpha]}\Theta(\Delta - \Delta_{[h\alpha]}) \simeq A\, e^{\Delta/T_H},$$

$$L_3(T) = \langle \mathrm{tr}_c e^{iA_0/T} \rangle = \frac{1}{2}\int d\Delta \frac{\partial n(\Delta)}{\partial \Delta} e^{-\Delta/T}.$$

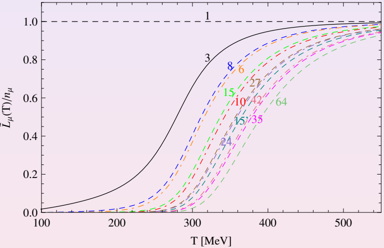
Issues

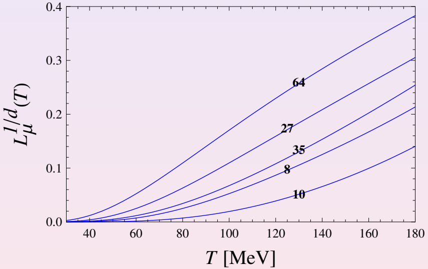
Polyakov loop in higher representations

[E.Megías, E. Ruiz Arriola, L.L. Salcedo, PRD89 '14]

$$\langle L_R \rangle = \frac{1}{Z} \int \mathcal{D}\Omega \, e^{-S_G - S_q} \, \mathrm{tr}(\Omega_R) \, .$$

$\Omega_R \equiv$ static source in R-representation.





Casimir scaling **[Karsch'85], [Schröder'99], [Kaczmarek'08], [Petreczky'15]**:

$$L_3(T) \sim (L_R(T))^{1/d_R} \qquad \text{with} \qquad d_R = C_2(R)/C_2(3) \, .$$

- Good approximation at $T > T_c$.
- Deviations at $T < T_c$.

Casimir scaling at low temperature

- Heavy-light relativistic system: Heavy source in rep R screened by a dynamical particle in representation $\bar{R}$. Hamiltonian:

$$H_R = p + \sigma_R r$$

- Eigenvalues: $H_R \psi_{n,\ell} = \varepsilon_{n,\ell}^{(R)} \psi_{n,\ell}$
- Polyakov loop in representation R:

$$L_R = e^{-F_R/T} = \sum_{n,\ell} g_{n,\ell}\, e^{-\varepsilon_{n,\ell}^{(R)}/T}$$

- Eigenvalues $\varepsilon_{n,\ell}^{(R)}$ scale like ($C_R \equiv$ quadratic Casimir invariant)

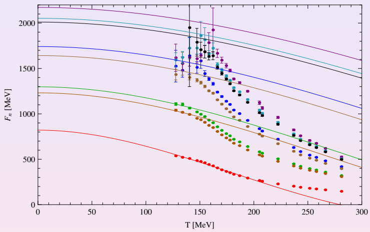
$$\varepsilon_{n,\ell}^{(R)} = \sqrt{\frac{\sigma_R}{\sigma_3}} \varepsilon_{n,\ell}^{(3)} , \qquad \text{and} \qquad \sigma_R = \frac{C_R}{C_3} \sigma_3 \, .$$

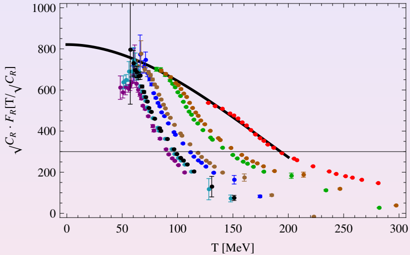
- Casimir scaling at low temperatures

$$\boxed{F_R(T) = \rho_R F_3(T/\rho_R) \, , \quad \text{where} \quad \rho_R = \sqrt{\frac{C_R}{C_3}}}$$

Casimir scaling at low temperature

Lattice data from [Petreczky, Schadler, PRD92 (2015)].





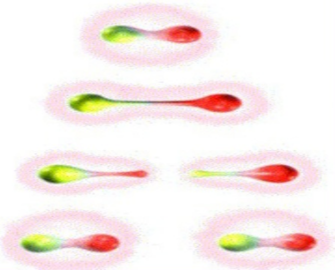
1. Fit of F_3 for $T < 200$ MeV.
2. Prediction for F_R by using Casimir scaling → Good agreement!!!

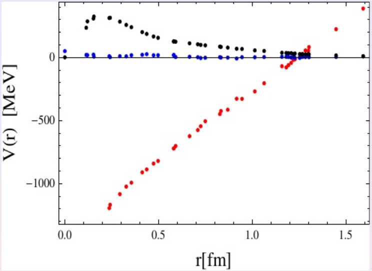
- Casimir scaling predicts universal behavior at low T → rescaled lattice data meet this curve.
- Note: H_R leads to $\frac{\varepsilon_{0,0}^{(R)}}{\sqrt{\sigma_R}} = 1.88 \Longrightarrow \varepsilon_{0,0}^{(3)} = 810\,\mathrm{MeV}$
  (good agreement with the fit of lattice data!!!)

Issues

Quark potential and String breaking

- Transition $Q\bar{Q} \to \mathrm{Meson}\,\overline{\mathrm{Meson}}$





- Energy of two heavy quarks: $E(r) = m_Q + m_{\bar{Q}} + V(r)$.
- Meson masses: $M_{Q\bar{q}} = \Delta_{Q\bar{q}} + m_Q\,, \quad M_{q\bar{Q}} = \Delta_{q\bar{Q}} + m_{\bar{Q}}$.
- Uncoupled Born-Oppenheimer (diabatic crossings)

$$V_{Q\bar{Q}}(r) = \sigma r\,, \qquad V_{(Q\bar{q})(q\bar{Q})}(r) = \Delta_{Q\bar{q}} + \Delta_{q\bar{Q}} \equiv 2\Delta\,.$$

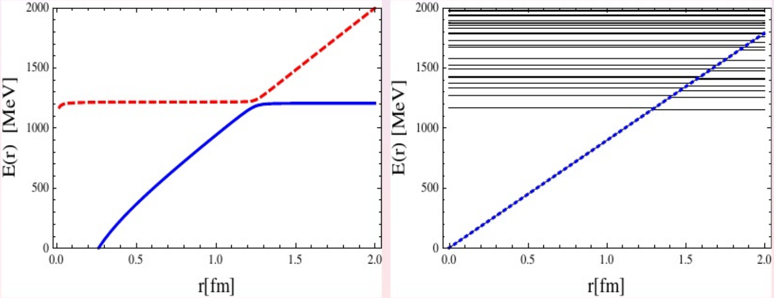
Excited states

- Estimate of the string breaking distance

$$V_{\bar{Q}Q}(r_c) = V_{(Q\bar{q})(q\bar{Q})}(r_c) \Longrightarrow \sigma r_c = 2M_{q\bar{Q}} - 2m_Q \sim 4M_0 \rightarrow r_c \simeq 1.2\,\mathrm{fm}$$

- In general many excited meson states

$$V^{(0,0)}_{Q\bar{Q}}(r) = \sigma r\,, \qquad V^{(n,m)}_{(Q\bar{q})(q\bar{Q})}(r) = \Delta^{(n)}_{Q\bar{q}} + \Delta^{(m)}_{q\bar{Q}}\,.$$

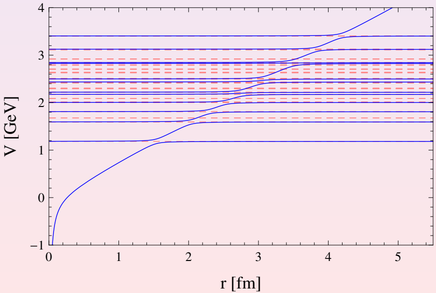


Avoided crossings

- Transition potential $V_{Q\bar{Q}\to M\bar{M}}(r)$, with coupled channels

$$V(r) = \begin{pmatrix} V_{Q\bar{Q}}(r) & V_{Q\bar{Q}\to M\bar{M}}(r) \\ V_{Q\bar{Q}\to M\bar{M}}(r) & V_{M\bar{M}}(r) \end{pmatrix} .$$

- After diagonalization ➞ Avoided crossing with states having the same quantum numbers as $Q\bar{Q}$:



Issues

Heavy $Q\bar{Q}$ free energy

- In the confined phase, the correlator between Polyakov loops becomes:

$$e^{-F_{Q\bar{Q}}(r,T)/T} = \langle \mathrm{Tr}_F \Omega(\vec{r}) \mathrm{Tr}_F \Omega(0)^\dagger \rangle = e^{-V_{Q\bar{Q}}(r)/T} + \sum_{n,m} e^{-V_{Q\bar{Q}}^{(n,m)}(r,T)/T} .$$

$F_{Q\bar{Q}}(r,T) \equiv$ heavy $Q\bar{Q}$ free energy.

- Neglecting the avoided crossing:

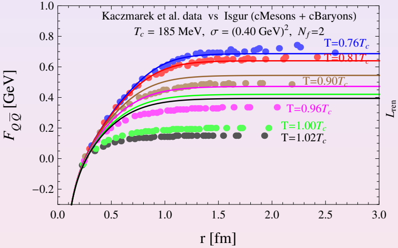
$$\boxed{e^{-F_{Q\bar{Q}}(r,T)/T} = e^{-V_{Q\bar{Q}}(r)/T} + \left(\frac{1}{2} \sum_n e^{-\Delta_n/T} \right)^2 , \quad \Delta_n \equiv \Delta_{Q\bar{q}}^{(n)} = \Delta_{q\bar{Q}}^{(n)}}$$

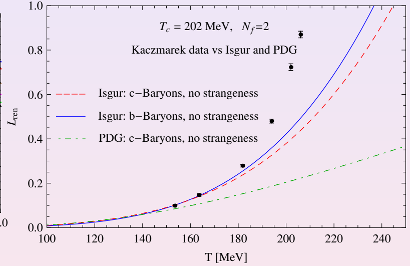
- Polyakov loop is computed as:

$$L(T) := \lim_{r \to \infty} e^{-F_{Q\bar{Q}}(r,T)/(2T)} = \frac{1}{2} \sum_n e^{-\Delta_n/T} \quad \equiv \text{HRG for Polyakov loop}$$

Heavy $Q\bar{Q}$ free energy

Lattice data: Kaczmarek, Zantow, PRD71 '05. $N_f = 2$





$$\boxed{e^{-F_{Q\bar{Q}}(r,T)/T} = e^{-V_{Q\bar{Q}}(r)/T} + \left(\frac{1}{2}\sum_n e^{-\Delta_n/T}\right)^2 , \quad \Delta_n \equiv \Delta^{(n)}_{Q\bar{q}} = \Delta^{(n)}_{q\bar{Q}}}$$

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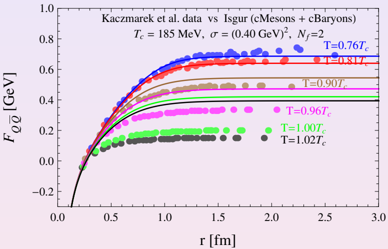
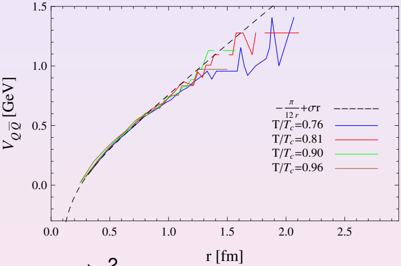


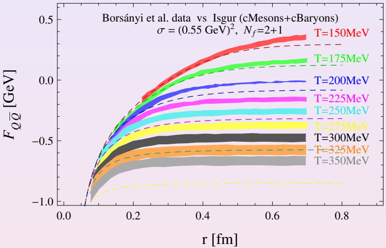


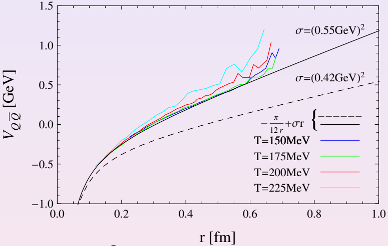
$$e^{-F_{Q\bar{Q}}(r,T)/T} = e^{-V_{Q\bar{Q}}(r)/T} + \left(\frac{1}{2}\sum_n e^{-\Delta_n/T}\right)^2, \qquad \Delta_n \equiv \Delta^{(n)}_{Q\bar{q}} = \Delta^{(n)}_{q\bar{Q}}$$

$$\rightarrow V_{Q\bar{Q}}(r) = -T\log\left[\underbrace{e^{-F_{Q\bar{Q}}(r,T)/T}}_{\text{From lattice}} - \underbrace{\left(\frac{1}{2}\sum_n e^{-\Delta_n/T}\right)^2}_{\text{From hadrom spectrum}}\right] \neq f(T)$$

Heavy $Q\bar{Q}$ free energy

Lattice data: Fodor et al. JHEP1504 '15. $N_f = 2+1$





$$e^{-F_{Q\bar{Q}}(r,T)/T} = e^{-V_{Q\bar{Q}}(r)/T} + \left(\frac{1}{2}\sum_n e^{-\Delta_n/T}\right)^2, \qquad \Delta_n \equiv \Delta^{(n)}_{Q\bar{q}} = \Delta^{(n)}_{q\bar{Q}}$$

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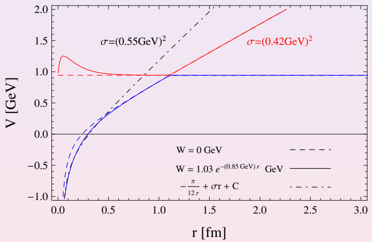
Issues

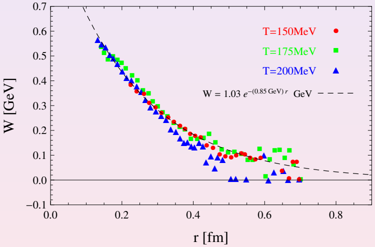
Avoided crossing

- Model with two states:

$$V(r) = \begin{pmatrix} -\frac{4\alpha}{3r} + \sigma r & W(r) \\ W(r) & 2\Delta \end{pmatrix} \qquad \sigma = (0.42\,\mathrm{GeV})^2 .$$

[Fodor et al. '15: N$_f = 2+1$]



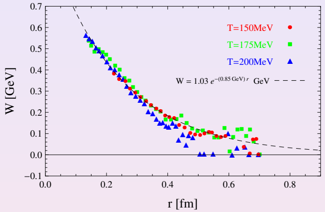


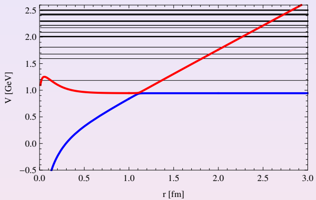
In qualitative agreement with lattice data of string breaking: **[Bali et al. PRD71 '05].**

$$W(r) = 1.03\, e^{-m\,r}\,\mathrm{GeV}\,, \qquad m = 0.85\,\mathrm{GeV}\,.$$

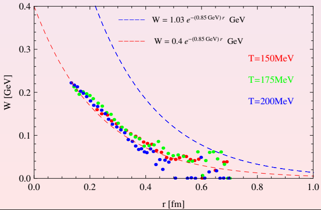
Avoided crossing

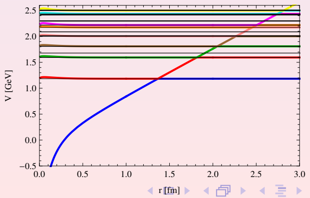
- Mixing: (string — lightest heavy-light meson)





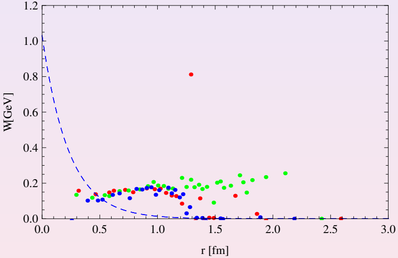
- Including more states $\Longrightarrow$ model for avoided crossing? $\Longrightarrow$ Mixing: string — heavy-light mesons with same quantum number





Avoided crossing

- What is the scale $m = 0.85\,\mathrm{GeV}$?
- With **[Kaczmarek et al. '05]**, $\mathbf{N}_f = 2$, it is not observed an appreciable avoided crossing. Why?



- **[Fodor]** and **[Kaczmarek]** use different renormalization prescriptions.
- **[Kaczmarek]** does not consider physical pion masses: $m_\pi \simeq 770$ MeV.

Thermodynamics of two heavy quarks

Heavy $Q\bar{Q}$ free energy ⟶ thermodynamics of two heavy quarks in a plasma:

- *Internal energy*

$$U_{Q\bar{Q}}(r,T) = F_{Q\bar{Q}}(r,T) - T\frac{\partial F_{Q\bar{Q}}(r,T)}{\partial T}$$

- *Entropy*

$$S_{Q\bar{Q}}(r,T) = -\frac{\partial F_{Q\bar{Q}}(r,T)}{\partial T}$$

- *Interaction measure (trace anomaly)*

$$\Delta_{Q\bar{Q}}(r,T) = U_{Q\bar{Q}}(r,T) + 3F_{Q\bar{Q}}(r,T) = 4F_{Q\bar{Q}}(r,T) - T\frac{\partial F_{Q\bar{Q}}(r,T)}{\partial T}$$

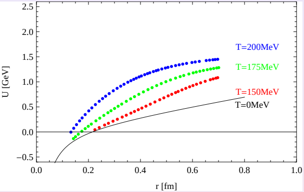
- *Specific heat (at constant volume)*

$$c_{V,Q\bar{Q}} = \left.\frac{\partial U_{Q\bar{Q}}}{\partial T}\right|_V$$

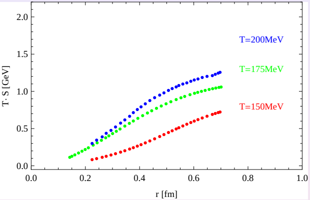
Using $F_{Q\bar{Q}}(r,T)$ ⟶ HRG description of $U_{Q\bar{Q}}$, $S_{Q\bar{Q}}$, $\Delta_{Q\bar{Q}}$ and $c_{V,Q\bar{Q}}$.

Thermodynamics of two heavy quarks

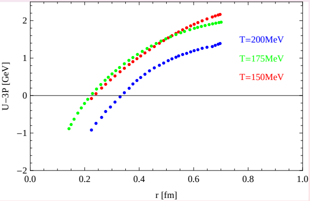
Internal Energy



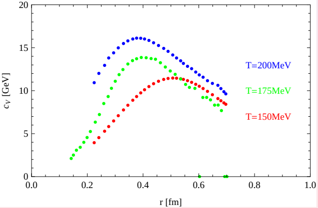
Entropy



Interaction measure



Specific heat



Results in qualitative agreement with **[O.Kaczmarek, EPJC61 (2009)].**

Conclusions:

- At low temperatures hadrons can be considered as a complete basis of states in terms of a Hadron Resonance Gas (HRG) model. The HRG works at $T \lesssim 0.8 T_c$.
- Close T_c many hadrons are needed to saturate the sum rule $\Longrightarrow$ What states are needed when approaching T_c from below?
- Chiral Quark Models coupled to the Polyakov loop have the right properties to study the transition from quarks and gluons to the hadronic degrees of freedom ➞ Quark-Hadron duality.
- In QCD: we derive a hadronic representation of the Trace Anomaly and Polyakov loop in terms of hadrons, glueballs, hybrid states, $\cdots$ ➞ HRG model for the Polyakov loop writes in terms of hadrons with 1 heavy quark.
- Hadronic representation for the heavy $Q\bar{Q}$ free energy leads to a very good agreement with lattice data.
- We studied the role played by avoided crossing in heavy $Q\bar{Q}$ free energy ➞ in agreement with lattice data of string breaking. More accurate lattice data and to lower T are welcomed.

Thank You!