Matrix formulation of quantum mechanics

most useful when we deal with finite discrete bases and then it follows matrix multiplication

$$|\psi\rangle = \sum_i c_i |u_i\rangle \qquad c_i = \langle u_i|\psi\rangle$$

$$\begin{pmatrix} \langle u_1|\psi\rangle \\ \langle u_2|\psi\rangle \\ \vdots \\ \langle u_i|\psi\rangle \end{pmatrix} = \begin{pmatrix} c_1 \\ c_2 \\ \vdots \\ c_i \\ \vdots \end{pmatrix}$$

kets are written as column vectors.

$$\langle\psi| = \sum_i c_i^* \langle u_i| \qquad c_i^* = \langle\psi|u_i\rangle$$

$$\left(\langle\psi|u_1\rangle \quad \langle\psi|u_2\rangle \quad \cdots \quad \langle\psi|u_i\rangle \cdots \right) = \left(c_1^* \quad c_2^* \cdots \quad c_i^* \cdots \right)$$

bras are written as row vectors

$$\hat{A} = \sum_{ij} A_{ij} |u_i\rangle\langle u_j| \qquad A_{ij} = \langle u_i|\hat{A}|u_j\rangle$$

$$\begin{pmatrix} A_{11} & A_{12} & \cdots & A_{1j} & \cdots \\ A_{21} & A_{22} & \cdots & A_{2j} & \cdots \\ \vdots & \vdots & & \vdots & \\ A_{i1} & A_{i2} & \cdots & A_{ij} & \cdots \\ \vdots & \vdots & & \vdots & \end{pmatrix}$$

operators are written as matrix.

Let's see some example:

$$\langle\psi|\varphi\rangle \Rightarrow \left.\begin{aligned} |\psi\rangle &= \sum_i c_i |u_i\rangle \\ |\varphi\rangle &= \sum_i d_i |u_i\rangle \end{aligned}\right\} \langle\psi|\varphi\rangle = \left(\sum_i c_i^* \langle u_i|\right)\left(\sum_j d_j |u_j\rangle\right)$$

$$\Rightarrow \sum_{ij} c_i^* d_j \langle u_i|u_j\rangle \Rightarrow \sum_i c_i^* d_i$$

scalar product for a row and a colum vector.

$$\begin{pmatrix} c_1^* & c_2^* & \cdots & c_i^* & \cdots \end{pmatrix} \begin{pmatrix} d_1 \\ d_2 \\ \vdots \\ d_i \\ \vdots \end{pmatrix} \Rightarrow c_1^* d_1 + c_2^* d_2 + \cdots + c_i^* d_i + \cdots$$

$$\sum_i c_i^* d_i$$