Representations in Quantum Mechanics

Choose the appropriate basis are good representation

Representation: an orthonormal basis provides a representation in state space. $\{|u_i\rangle\}$ $\langle u_i|u_j\rangle = \delta_{ij}$.

- $\{|u_i\rangle\}$ forms an orthonormal basis if for every ket $|\psi\rangle \in V$ there is a unique expansion

$$|\psi\rangle = \sum_i c_i |u_i\rangle$$

expansion coeffs

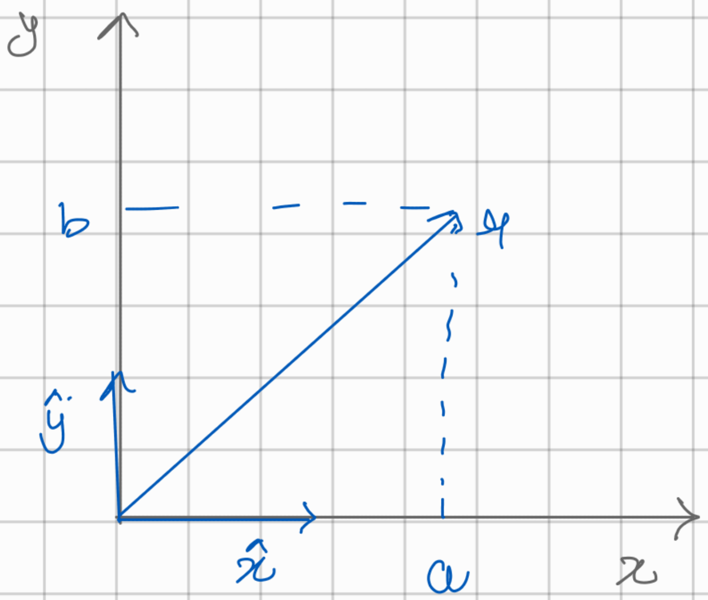
Project the state $|\psi\rangle$ on the basis ket u_j

$$\langle u_j|\psi\rangle = \langle u_j|\left(\sum c_i |u_i\rangle\right) \Rightarrow \sum c_i \langle u_j|u_i\rangle$$

$$\Rightarrow \sum c_i \delta_{ij}$$

$$\Rightarrow c_j$$

- $\{c_i\}$ are the representation of $|\psi\rangle$ in the $\{|u_i\rangle\}$ basis.



$\{|u_i\rangle\}$ $\langle u_i|u_j\rangle = \delta_{ij}$

$$\{\hat{x}, \hat{y}\} \Rightarrow \begin{cases} \hat{x}\cdot\hat{x} = 1 \\ \hat{x}\cdot\hat{y} = 0 \\ \hat{y}\cdot\hat{y} = 1 \end{cases}$$

$$|\psi\rangle = \sum c_i |u_i\rangle$$

$$c_i = \langle u_i|\psi\rangle$$

$r = a\hat{x} + b\hat{y}$

$\hat{x}\cdot r = a\underbrace{\hat{x}\cdot\hat{x}}_{1} + b\underbrace{\hat{x}\cdot\hat{y}}_{0} \Rightarrow a$

similarly $\hat{y}\cdot r = b$

Projection of the vector on the basis vector.

Closure relation

$$|\psi\rangle = \sum_i \underbrace{\langle u_i|\psi\rangle}_{\text{scalar}} |u_i\rangle \Rightarrow \sum_i |u_i\rangle\langle u_i|\psi\rangle$$

$$\Rightarrow \underbrace{\left(\sum_i |u_i\rangle\langle u_i|\right)}_{\text{operator written as an outer product.}} |\psi\rangle$$

but what does this operator do? Identity operator.

$$\mathbb{1} = \sum_i |u_i\rangle\langle u_i|$$

~ closure relation or the resolution of the identity.

$\langle\psi| \in V^*$ $\qquad \langle\psi| = \langle\psi|\mathbb{1}$

$$\langle\psi| = \langle\psi|\left(\sum_i |u_i\rangle\langle u_i|\right)$$

$$\Rightarrow \sum_i \underbrace{\langle\psi|u_i\rangle}_{\text{scalar}}\langle u_i|$$

$$\langle\psi|u_i\rangle = \langle u_i|\psi\rangle^* = c_i^*$$

$$\langle\psi| = \sum_i c_i^* \langle u_i| \qquad c_i^* = \langle\psi|u_i\rangle$$

Representation of Operators

$$\hat{A}|\psi\rangle = |\psi'\rangle$$

$$c_i = \langle u_i|\psi\rangle$$

$$\hat{A}\sum_i c_i|u_i\rangle = \sum_i c_i'|u_i\rangle$$

$$c_i' = \langle u_i|\psi'\rangle = \langle u_i|\hat{A}|\psi\rangle \Rightarrow \langle u_i|\hat{A}\,\mathbb{1}|\psi\rangle$$

$$\Rightarrow \langle u_i|\hat{A}\Big(\sum_j |u_j\rangle\langle u_j|\Big)|\psi\rangle$$

$$c_i' \Rightarrow \sum_j \langle u_i|\hat{A}|u_j\rangle\langle u_j|\psi\rangle$$

$$|\psi'\rangle = \sum_i\Big(\sum_j \langle u_i|\hat{A}|u_j\rangle\langle u_j|\psi\rangle\Big)|u_i\rangle$$

$$\Rightarrow \sum_{ij}\Big(|u_i\rangle\langle u_i|\,\hat{A}\,|u_j\rangle\langle u_j|\Big)|\psi\rangle$$

$$\hat{A} = \sum_{ij} A_{ij}|u_i\rangle\langle u_j| \qquad \text{where } A_{ij} = \langle u_i|\hat{A}|u_j\rangle$$

Discreete to continuous

$$\{|u_i\rangle\} \Longrightarrow \{|v_\alpha\rangle\}$$

$$\langle u_i|u_j\rangle = \delta_{ij} \Longrightarrow \langle v_\alpha|v_\beta\rangle = \delta(\alpha-\beta)$$

Kronecker function   Dirac Delta function

$$|\psi\rangle = \sum_i c_i|u_i\rangle \Longrightarrow |\psi\rangle = \int d\alpha\, c(\alpha)\,|v_\alpha\rangle$$

$c_j = \langle u_j | \psi \rangle$

$c(\beta) = \langle v_\beta | \psi \rangle$