Circuit knitting with classical communication

For circuit containing n local CNOT gates connecting two circuit parts, the simulation overhead can be reduced from $O(9^n)$ to $O(4^n)$ if one allows for classical information exchange.

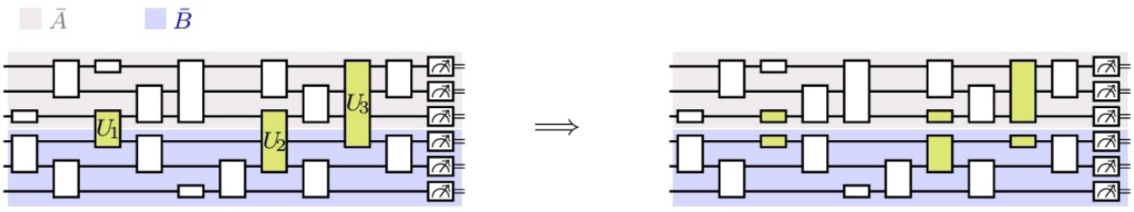


Figure 1: The nonlocal circuit on the left can be simulated with local circuits on the right using quasiprobability simulation. If the optimal quasiprobability simulation is performed on each gate U_1, U_2, U_3 individually, then the total simulation overhead is given by $\gamma_S(U_1)^2\gamma_S(U_2)^2\gamma_S(U_3)^2$, as described in the main text.

i) Local operation (LO) The two computers can only realize operations in a product form $A \otimes B$ where A and B act locally on $\bar{A}$ and $\bar{B}$

ii) Local operations and one way classical communication ($LO\vec{CC}$) : The two computers can realize protocols that contain local operation from (LO) as well as classical communication from $\bar{A}$ to $\bar{B}$.

iii) Local operations and classical communications (LOCC) : The two computers can realize protocols that contain local operations from LO as well as two-way communication b/w $\bar{A}$ and $\bar{B}$

For LO and $LO\vec{CC}$ one does not require two separate quantum computers. Instead one can run the two subcircuits in sequence on the same device.

The classical communication in the $LO\vec{CC}$ setting can be simulated by classically storing the bits sent from $\bar{A}$ to $\bar{B}$. Only LOCC requires two separate quantum computers.

Quasi probability Simulation

$$\mathcal{U} = \sum_i a_i F_i$$

$a_i \in \mathbb{R}$ and can be negative

During the circuit execution, the gate $\mathcal{U}$ gets randomly replaced by one of the gates F_i. The sampling overhead of the quasi probability simulation is given by K^2 where $K := \sum_i |a_i|$

The smallest achievable K for a gate U described in the setting $S \in \{LO, LO\vec{CC}, LOCC\}$ is denoted by $\gamma_S(U)$

$$\gamma_{LOCC}(U) \le \gamma_{LO\vec{CC}}(U) \le \gamma_{LO}(U)$$

when simulating a single non-local gate U via optimal QPD, the number of samples required to achieve a fixed accuracy increases by $\gamma_S(U)^2$.

If the circuit contains n non local gates, the overall sampling overhead is given by

$$\prod_{i=1}^{n} \gamma_S(U_i)^2$$ which scales as $\exp(O(n))$

which emphasizes that the number of non local gates 'n' may not be too large.

How does classical communication help?

No advantage of rotations

For large class of two qubit unitaries U, including all Clifford gates like the controlled-Rotations $CR_X(\theta)$, $CR_Y(\theta)$, $CR_Z(\theta)$ and two qubit rotations $R_{XX}(\theta)$, $R_{YY}(\theta)$, $R_{ZZ}(\theta)$ there is no advantage in having classical communication when QPD simulating a single instance of the gate with local operation,

$$\gamma_{LOCC}(U) = \gamma_{LO\vec{CC}}(U) = \gamma_{LO}(U)$$

but thing changes as we consider a circuit with many instances of the same non-local gate.

$$\gamma_{LOCC}(U)^{2n} \longrightarrow \gamma^{(n)}_{LOCC}(U)^{2n}$$

$$\text{where } \gamma^{(n)}_{LOCC}(U) = \gamma_{LOCC}(U^{\otimes n})^{1/n}$$

Also, utilizing the optimal quasi probability simulation for each non-local gate individually is not optimal.

Circuit knitting using quasiprobability simulation

Two quantum system A and B. $L(A) \sim$ linear maps from $A \to A$

$\mathcal{L}(A): L(L(A)) \sim$ linear superop. from A to A.

$CPTN(A) \subset \mathcal{L}(A)$

set of completely positive trace non-increasing maps acting on A to itself.

$$D(A) := \{ \mathcal{E} \in \mathcal{L}(A) \mid \exists\, \mathcal{E}^+, \mathcal{E}^- \in CPTN(A) \text{ s.t. } \mathcal{E} = \mathcal{E}^+ - \mathcal{E}^- \text{ and } \mathcal{E}^+ + \mathcal{E}^- \in CPTN(A) \}$$

The set of local operations $LO(A,B)$ is defined as all maps on $A \otimes B$ in product form $A \otimes B$, where $B \in D(B)$ and $A \in D(A)$

Consider a quantum circuit on which we want to apply quasiprobabilistic circuit knitting. We first group the involved qubits into two systems, denoted $\bar{A}$ and $\bar{B}$. For simplicity we consider the case, where there is only one single non local gate U, acting on both $\bar{A}$ and $\bar{B}$

$$\mathcal{G}_2 \circ \mathcal{U} \circ \mathcal{G}_1 \qquad\qquad O_{\bar{A}} \otimes O_{\bar{B}}$$

unitary channel corresponding to U and $\mathcal{G}_j$ only act locally on $\bar{A}$ and $\bar{B}$.

measurement done

We want to determine

Expectation value: $\mathrm{tr}\big[(O_{\bar{A}} \otimes O_{\bar{B}})\, \mathcal{G}_2 \circ \mathcal{U} \circ \mathcal{G}_1 (|0\rangle\langle 0|_{\bar{A}\bar{B}}) \big]$

measurement operator. quantum channel initial state

QPD

$$\mathrm{tr}\big[(O_{\bar{A}} \otimes O_{\bar{B}})\, \mathcal{G}_2 \circ \mathcal{U} \circ \mathcal{G}_1 (|0\rangle\langle 0|_{\bar{A}\bar{B}}) \big] = \sum_i p_i \, \mathrm{tr}\big[(O_{\bar{A}} \otimes O_{\bar{B}})\, \mathcal{G}_2 \circ \mathcal{F}_i \circ \mathcal{G}_1 (|0\rangle\langle 0|_{\bar{A}\bar{B}}) \big]\, K \,\mathrm{sign}(a_i)$$

$K = \sum_i |a_i|$ and $p_i = |a_i| / K$ prob. distribution.

make use of Monte Carlo sampling.

For each shot of the circuit, we randomly replace the non local gate $\mathcal{U}$ with one of the local gates $\mathcal{F}_i$ with probability p_i.
The measurement is weighted by $K\,\mathrm{sign}(a_i)$. By repeating this procedure many times and averaging the result, we can get an arbitrary good estimate of the desired quantity $\propto K^2$

Can be generalized to 'n' non-local gates. $\propto \prod_{i=1} K_i$

We want to minimize this, hence we make use of QPDs with the smallest possible sampling overhead. Such QPDs are therefore called the optimal QPDs. We call the smallest achievable value γ-factor.

The γ-factor of $\mathcal{E} \in \mathcal{L}(A \otimes B)$ over $S \in \{LO(A,B), \overrightarrow{LOCC}(A,B), LOCC(A,B)\}$ is defined as

$$\gamma_S(\mathcal{E}) := \min\left\{ \sum_{i=1}^{m} |a_i| : \mathcal{E} = \sum_{i=1}^{m} a_i F_i \text{ where } m \geq 1, F_i \in S \text{ and } a_i \in \mathbb{R} \right\}$$

$\gamma_{LO}(\mathcal{E}) > \gamma_{\overrightarrow{LOCC}}(\mathcal{E}) > \gamma_{LOCC}(\mathcal{E})$ also we write $\gamma_S(U)$ as the γ-factor of unitary channel induced by U.

The γ-factor is submultiplicative under the tensor product.

Local quasiprobability decompositions for states

ρ_{AB}: The γ-factor of ρ_{AB} over S is

$$\gamma_S(\rho_{AB}) = \min\left\{ \gamma_S(\mathcal{E}) : \mathcal{E} \in \mathcal{L}(A \otimes B) \text{ s.t. } \mathcal{E}(|0\rangle\langle 0|)_{AB} = \rho_{AB} \right\}$$

This quantity characterizes the optimal sampling overhead required to generate the bipartite state ρ_{AB} using quasiprobabilistic circuit knitting. Classical communication does not change the sampling overhead for the task of state preparation.

The overhead of simulating n non local CNOT gates is reduced from $O(9^n)$ to $O(4^n)$ since

$\gamma_{LOCC}(CNOT) = 3$ but $\gamma_{LOCC}^{(n)}(CNOT) = (2^{n+1} - 1)^{1/n}$

$$r_{LOCC}^{(n)}(CNOT)^{2n} = O(4^n)$$

Realize the desired unitary via Gate teleportation.

Quantum Gate teleportation is the act of being able to apply a quantum gate on the unknown state while it's being teleported. (power of entangling measurement)

Quantum teleportation uses measurement to transfer quantum information from one place to another.

C-NOT b/w two qubits using Quantum Teleportation

Quantum teleportation is a universal computational primitive. We can create variety of interesting gates by teleporting quantum bits through special entangled states.

A quantum computer can be constructed using just single quantum bit operations, <u>bell basis measurements</u> and <u>Greenberger - Horne - Zeillinger states</u>.

Bell Measurement is a joint measurement performed on 2-qubit state. whose result is always a projection onto a Bell state.

Bell state : A two qubit state with maximal entanglement

Bell Basis	2 qubit	$\|00\rangle$	$\|01\rangle$	$\|10\rangle$	$\|11\rangle$
		$\|0\rangle$	$\|1\rangle$	$\|2\rangle$	$\|3\rangle$

in bell basis set

$$|\psi^{\pm}\rangle = \frac{1}{\sqrt{2}}(|00\rangle \pm |11\rangle) = \frac{1}{\sqrt{2}}\begin{bmatrix}1\\0\\0\\\pm1\end{bmatrix} \qquad |\phi^{\pm}\rangle = \frac{1}{\sqrt{2}}(|01\rangle \pm |10\rangle) = \frac{1}{\sqrt{2}}\begin{bmatrix}0\\1\\\pm1\\0\end{bmatrix}$$

Any two qubit state can be expressed in any basis. A measurement in a particular basis has eigenvectors in that set, and a measurement will observe the eigenvalue of one

Measurement is an interface between the quantum and classical world. and is generally irreversible (i.e it destroys the state)

In certain carefully designed cases, like quantum teleportation uses measurement to transfer quantum information from one place to another.

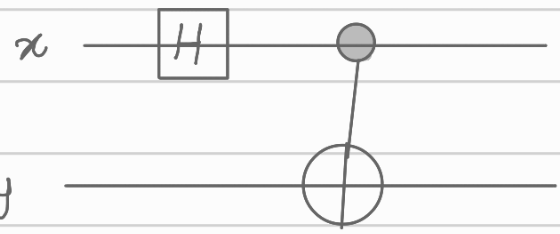
Quantum Information is preserved only in a subspace of the measured system. By selecting our initial state to lie in this preserved subspace we can ensure, paradoxically, that the measurement tells us nothing about the quantum data.

Unitary Transforms By Teleportation

a) How Quantum Teleportation works?

$$|\alpha\rangle = a|0\rangle + b|1\rangle$$

$$|\psi\rangle = \frac{(|00\rangle + |11\rangle)}{\sqrt{2}}$$



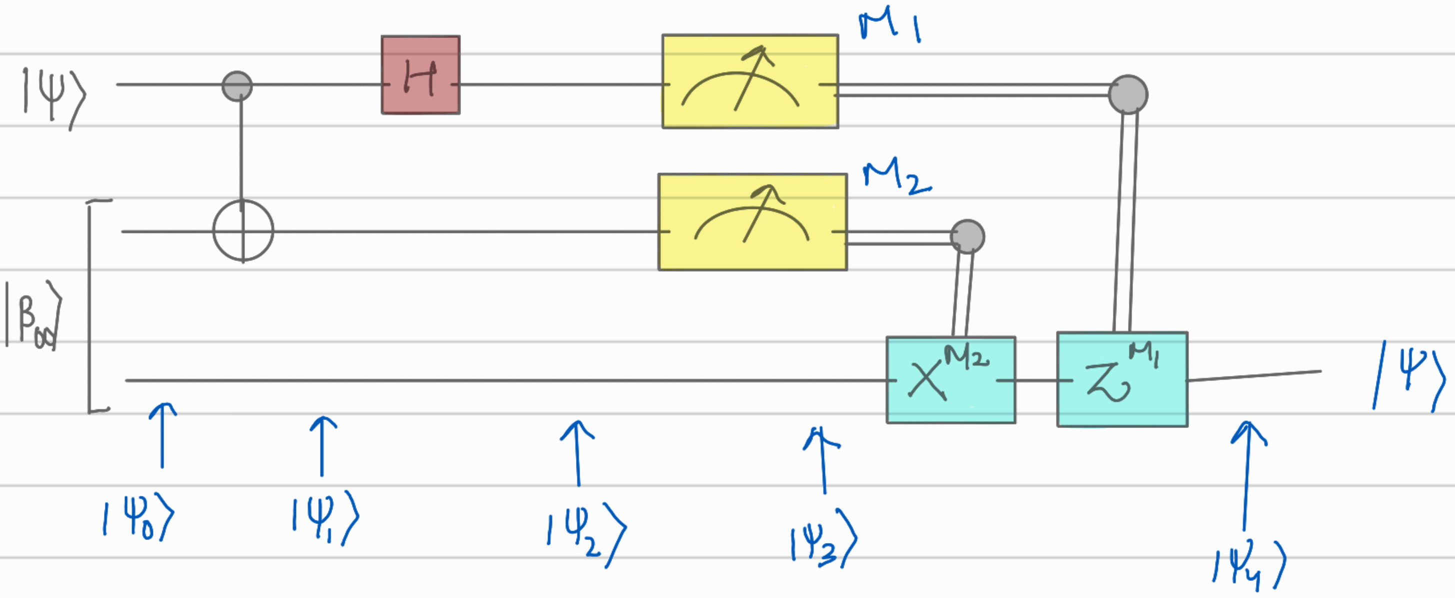
In	Out
$\lvert 00\rangle$	$(\lvert 00\rangle + \lvert 11\rangle)/\sqrt{2} \equiv \lvert\beta_{00}\rangle$
$\lvert 01\rangle$	$(\lvert 01\rangle + \lvert 10\rangle)/\sqrt{2} \equiv \lvert\beta_{01}\rangle$
$\lvert 10\rangle$	$(\lvert 00\rangle - \lvert 11\rangle)/\sqrt{2} \equiv \lvert\beta_{10}\rangle$
$\lvert 11\rangle$	$(\lvert 01\rangle - \lvert 10\rangle)/\sqrt{2} \equiv \lvert\beta_{11}\rangle$

These states are known as Bell states, or sometimes the EPR states or EPR pairs, after Bell, and Einstein, Podolsky, and Rosen

$$|\beta_{xy}\rangle \equiv |0,y\rangle + (-1)^x|1,\bar{y}\rangle$$

$\sqrt{2}$

Quantum teleportation is a technique for moving quantum states around, even in the absence of a quantum communications channel linking the sender of the quantum state to the recipient.



$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$

The state input into the circuit $|\psi_0\rangle$ is

$$|\psi_0\rangle = |\psi\rangle|\beta_{00}\rangle$$

$$\Rightarrow \frac{1}{\sqrt{2}}\left[\underset{A}{\alpha|0\rangle}\left(\underset{AB}{|00\rangle} + \underset{AB}{|11\rangle}\right) + \beta|1\rangle\left(|00\rangle + |11\rangle\right)\right]$$

the first two qubits (on the left) belong to Alice, and the third qubit to Bob. Alice's second qubit and Bob's qubit start out in an EPR state.

Alice sends her qubit through a CNOT gate, obtaining

$$\Rightarrow \frac{1}{\sqrt{2}}\left[\alpha|0\rangle\left(|00\rangle + |11\rangle\right) + \beta|1\rangle\left(|10\rangle + |01\rangle\right)\right]$$

She then sends the first qubit through a Hadamard gate

$$\Rightarrow \frac{1}{2}\left[\alpha\left[|0\rangle+|1\rangle\right]\left[|00\rangle+|11\rangle\right]+\beta\left[|0\rangle-|1\rangle\right]\left[|10\rangle+|01\rangle\right]\right]$$

Possible states for BOB

Regrouping

$$|\psi_2\rangle = \frac{1}{2}\Big[\underset{A}{|00\rangle}\underset{B}{\boxed{(\alpha|0\rangle+\beta|1\rangle)}} + |01\rangle\boxed{(\alpha|1\rangle+\beta|0\rangle)} + |10\rangle\boxed{(\alpha|0\rangle-\beta|1\rangle)} + |11\rangle\boxed{(\alpha|1\rangle-\beta|0\rangle)}\Big]$$

This expression naturally breaks down into four terms. The first term has Alice's qubits in the state $|00\rangle$, and Bob's qubit in the state $\alpha|0\rangle+\beta|1\rangle$ — which is the original state of $|\psi\rangle$.

If Alice performs a measurement and obtains 00, then Bob's system will be in $|\psi\rangle$.

Depending on Alice's measurement outcome, Bob's qubit will end up in one of these four possible states.

Of course, to know which state it is in, Bob must be told the result of Alice's measurement, and this prevents teleportation from being used to transmit information faster than light.

For $|00\rangle$ $\rightarrow$ Bob does nothing

For $|01\rangle$ $\longrightarrow$ Apply X

For $|10\rangle$ $\longrightarrow$ Apply Z

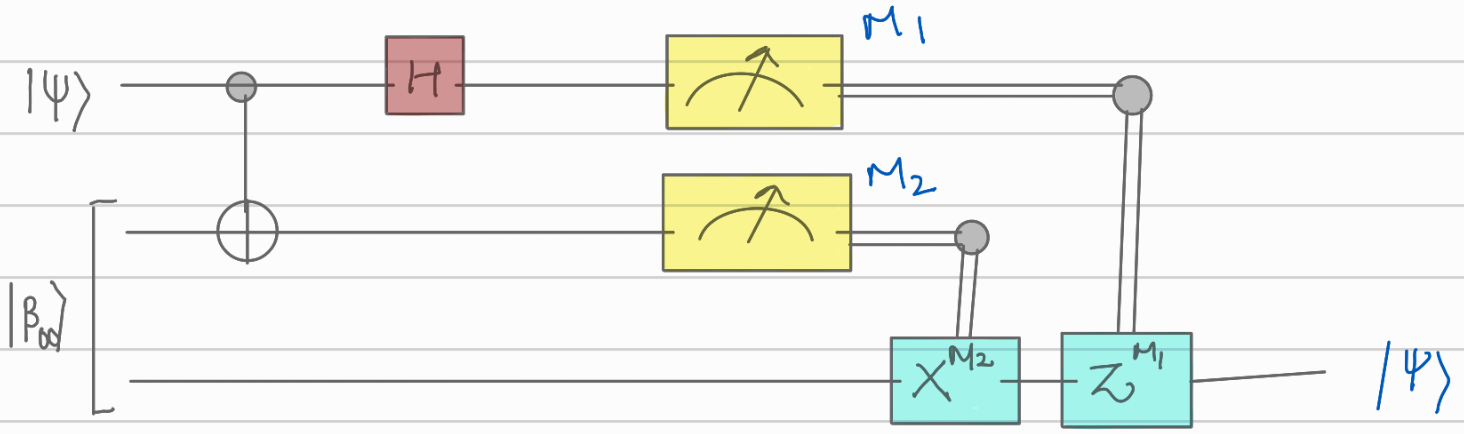
For $|11\rangle$ ⟶ Apply X and then Z

$$Z^{M_1} X^{M_2}$$

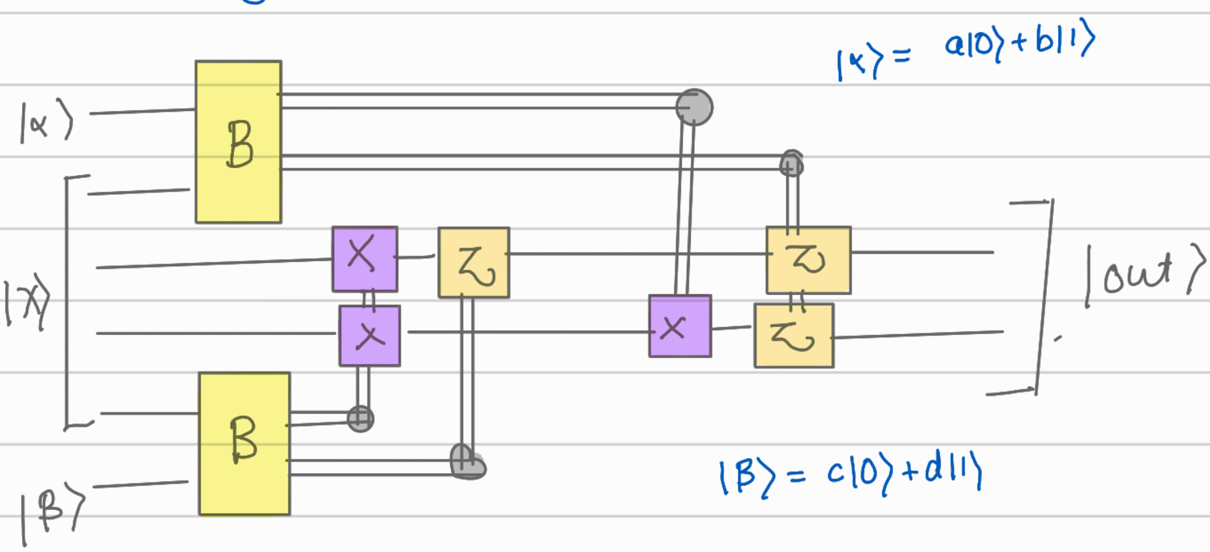
Theory of relativity implies that faster than light information transfer could be used to send information backwards in time.

without the classical communication, teleportation does not convey any information at all. The classical channel is limited by the speed of light.

For this work, we simply reverse the appropriate Pauli operator to reconstruct $|\alpha\rangle$. Replication of this circuit allows teleportation of multiple qubits.



The same basic idea can be used to teleport two qubits through a CNOT gate.

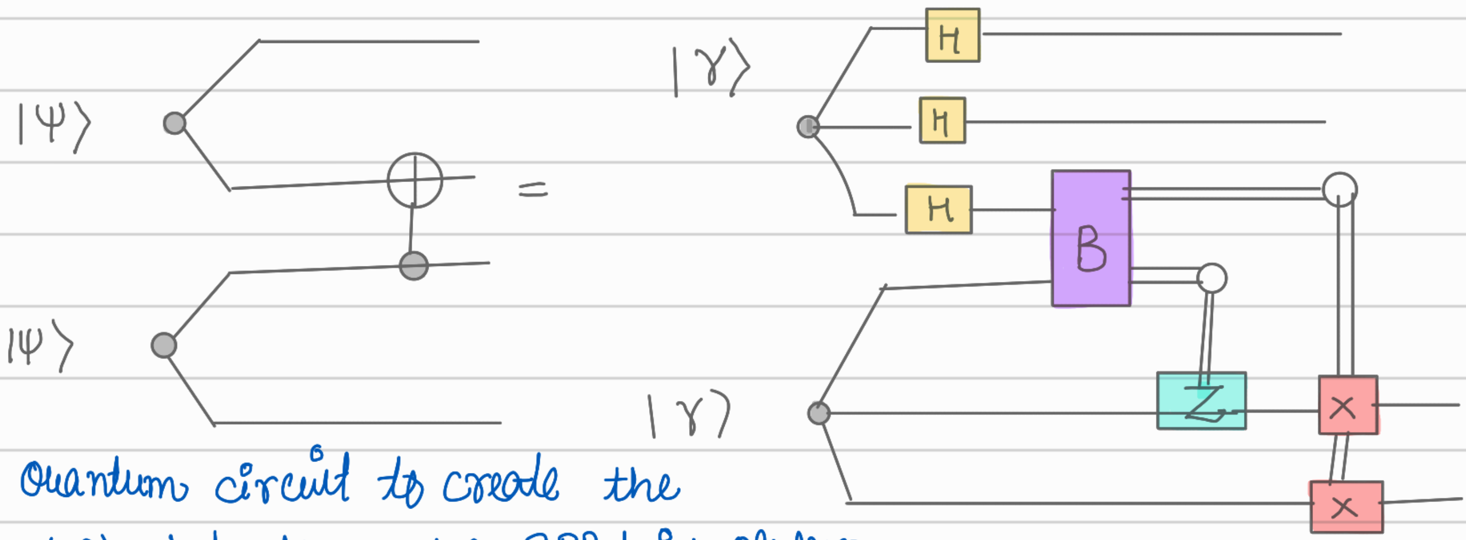


$$|\chi\rangle = (|00\rangle + |11\rangle)|00\rangle + (|01\rangle + |10\rangle)|11\rangle$$

[illegible]

$$\frac{[\text{illegible}]}{\sqrt{2}}$$

$|\beta\rangle$ is the control and $|\alpha\rangle$ is the target.



Quantum circuit to create the $|\alpha\rangle$ state from two EPR pairs, or from two GHZ states

$$|\gamma\rangle = (|000\rangle + |111\rangle)/\sqrt{2}$$

Circuit knitting with Classical Communications

For a large class of two qubit unitaries (CR_X, CR_Y, CR_Z, R_{XX}, R_{YY}, R_{ZZ}) there is no advantage in having classical communication when quasiprobability simulating a single instance of the gate with local operations.

$$\gamma_{LOCC}(U) = \gamma_{LO\vec{CC}}(U) = \gamma_{LO}(U)$$

However the statement only asserts that classical communication do not provide any advantage for simulating a single instance of the gate U.

For n-instances for some nonlocal Clifford gate U from $\gamma_{LOCC}(U)^{2n}$ to $\gamma^{n}_{LOCC}(U)^{2n}$ where

$$\gamma^{(n)}_{LOCC}(U) = \gamma_{LOCC}(U^{\otimes n})^{1/n}$$

also $\gamma^{(n)}_{LOCC}(U) < \gamma_{LOCC}(U)$

Utilizing the optimal qpd simulation for each non local gate individually is not optimal.

Eg:

$O(9^n)$ to $O(4^n)$ for n nonlocal CNOT

$\gamma_{LOCC}(CNOT) = 3$

$$\gamma^{(n)}_{LOCC}(CNOT) = (2^{n+1} - 1)^{1/n}$$

$$\gamma^{(n)}_{LOCC}(CNOT)^{2n} = O(4^n)$$

Circuit knitting could be simulated classically, hence cannot be used as a stand alone to demonstrate quantum advantage.