Globally Optimal Quantum Control

Control problems are notoriously difficult because they are non-convex and plagued with local extrema. Current optimization method must be repeated many times to find a good solutions.

Quantum Control via Polynomial Optimization

this method directly finds the global optimal solution.

What is Quantum Control?

It is generally concerned with manipulation of the evolution of quantum dynamical system towards a suitable defined optimum.

Constrained optimization problems are known to be highly non-smooth and non-convex and plagued by many local extrema

Popular quantum control algorithms such as GRadient Ascent Pulse Engineering (GRAPE), Chopped RAndom Basis (CRAB) are local. Hence they tend to find trap rather than desired global minimum.

This work is on Quantum Control via Polynomial Optimization that guarantees globally optimal solutions even for severly constrained problems.

Key Idea: The key idea is to reformulate a quantum control problem as a polynomial optimization that can be solved by very powerful global algorithms.

Quantum control is a discipline that addresses the following question: How

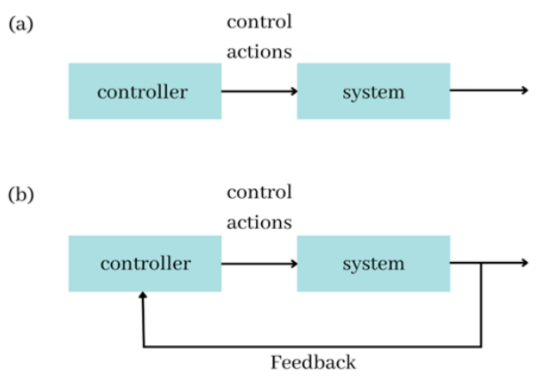
Quantum control is a discipline that addresses the following question; How can systems that obey the laws of quantum mechanics be efficiently manipulated to create desired behaviors? Ultimately, quantum control is concerned with how the classical world interacts with quantum devices. It guides researchers in gaining information about system dynamics through measurements and enabling useful performance in computing, sensing, and metrology.

Open-Loop Control

In open-loop control theory, the series of operational instructions do not rely on any measurement. This can be thought of as a linear, one-directional operation for simplicity. As shown in figure 1, the controller specifies a set of operational protocols that need to be executed for the system to function as desired.

The quantum mechanical version of open-loop control closely obeys the operational principles of its classical counterpart. The similarities in both quantum and classical, open-loop control methods are that they do not need to be updated with a measurement from the system.

Some variations of open-loop strategies for quantum systems are variable control, Lyapunov-based control, and incoherent control.



Closed-Loop Control

Closed-loop control requires a measurement of the system being controlled. As shown in figure 1(b), measurements are carried out on the system which are actively fed back to the controller. The controller uses the data acquired from the measurements to adjust different parameters. A closed-loop controller uses the feedback data to augment the system's output.

When closed-loops control techniques are applied to quantum systems, noise is introduced. Many control algorithms have been developed to characterize the noise contributions in quantum control. Closed-loop learning control, quantum filtering, and direct and indirect feedback control are some of the strategies used to mitigate noise in quantum control.

Optimal quantum Control

Unitary evolution operator on $\mathcal{H}$ (Hilbert space)

$$\partial_t U(t) = A(t) U(t)$$

where the anti-Hermitian operator $A(t)$ is defined as

$$A(t) = -i(H_0 + E(t)V)$$

when we shift the system's time from $t=0$ to $t=1$, we can define the following operator

$$\hat{U} = e^{-iHt/\hbar}$$

(must be norm preserving and invertible)

$$\hat{H} = H^{\dagger} \quad \text{(Hamiltonian)}$$

From Schrodinger's equation

$$i\hbar \partial_t \Psi = \hat{H}\Psi$$

$$\partial_t U(t) = A(t)\,U(t)$$

where

$$A(t) = -i(H_0 + E(t)V)$$

drift Hamiltonian (H_0) — control Hamiltonian ($E(t)V$)

$E(t) : [0,T] \to \mathbb{R}$ control field

Setting $\hbar = 1$ and the initial condition of $U(0) = \mathbb{1}$

The task for terminal coherent quantum control is to find control $E(t)$ such that for a given terminal time T we approach the target unitary U.

$$\underset{E(t)}{\text{minimize}} \; \| U(T) - U^* \|_F^2$$

$$\text{subj to} \quad \partial_t U(t) = A(t)\,U(t)$$

$$U(0) = \mathbb{1}$$

A more robust formulation is to maximize the Hilbert-Schmidt [illegible] overlap b/w the target and synthesized unitary

$$\max_{E(t)} \left| \mathrm{Tr}\left(U(T)^{\dagger} U^{*} \right) \right|^2$$

subs to

$$\partial_t U(t) = A(t) U(t)$$

$$U(0) = \mathbb{1}$$