Overhead-constrained circuit knitting for variational quantum

An approach that addresses challenges in simulating big quantum circuit by employing circuit-knitting to partition a large quantum system into smaller subsystems that can be simulated on a separate device.

Hybrid quantum-classical computing approaches allow for quantum simulations on a larger scale.

Focus of the next generation of quantum processors lies in connecting multiple medium-size quantum chips, allowing for parallelization of quantum simulation with real-time classical communication

The entanglement between the partitions should be weak such that classical methods can be efficiently employed to recombine the subsystems.

This work, proposes a method for quantum time evolution that splits a quantum circuit ansatz into multiple subsystems using circuit knitting, while keeping the sampling overhead controlled, by imposing a constraint on the circuit parameters during the optimization of variational quantum circuit.

With a realistic sampling overhead, we can significantly improve the accuracy of the simulation compared to a pure block product approximation, which does not consider any entanglement between different blocks.

The trade-off between the sampling overhead and the accuracy of the variational state can be tuned in a controllable way via a single hyperparameter of the optimization.

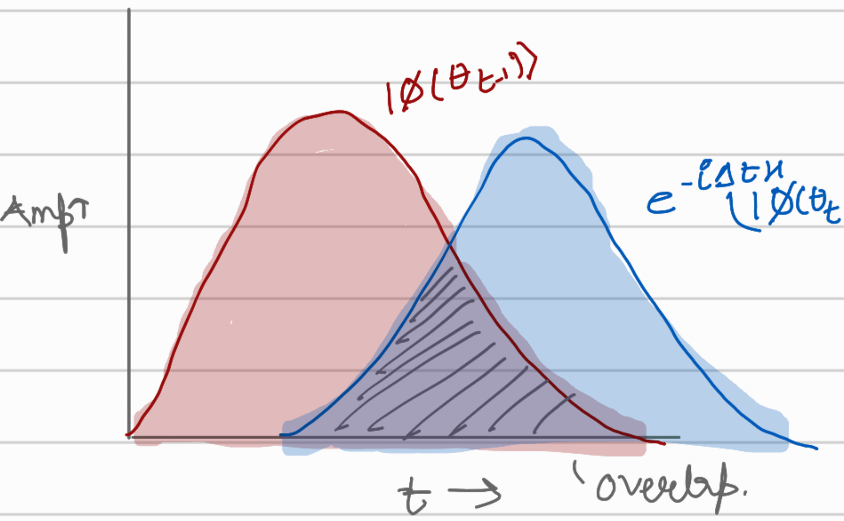
Projectional Variational Quantum Dynamics

A quantum algorithm for real-time evolution. It's a variational algorithm that projects the state at time $t + \Delta t$ onto a parameterized quantum circuit.

For a quantum state $|\phi(\theta)\rangle = U(\theta)|0\rangle$ constructed using a parameterized quantum circuit $U(\theta)$ and a Hamiltonian H, the update rule is:

$$\theta_{n+1} = \theta_n + \underset{\delta\theta}{\arg\min}\left[1 - |\langle\phi(\theta_n+\delta\theta)|\, e^{-i\Delta_t H}\, |\phi(\theta_n)\rangle|^2\right] \quad \text{— (i)}$$

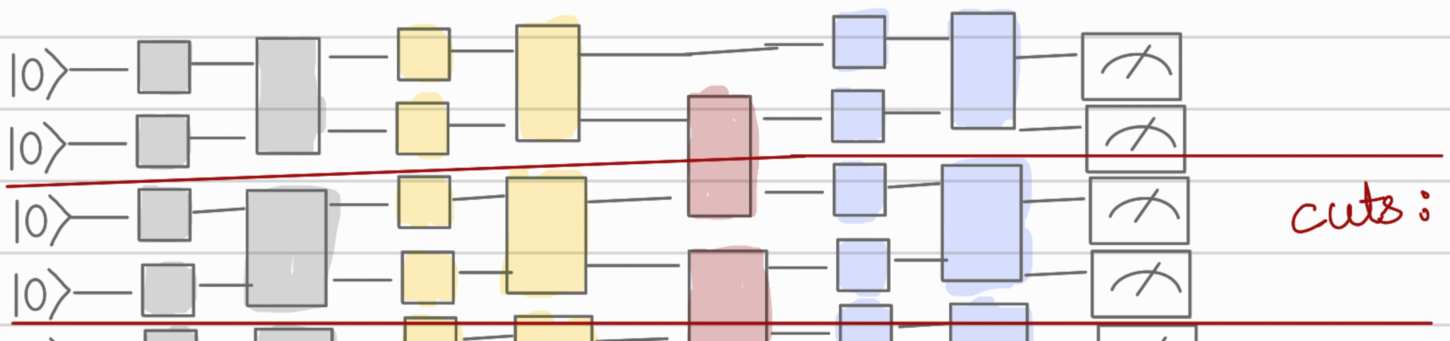
For small time-steps, it is not affected by barren plateaus, as the initial guess $|\phi(\theta_{t-1})\rangle$ has a non zero overlap with the target state $e^{-i\Delta t H}|\phi(\theta_{t-1})\rangle$.



state evolution in PVQD

Observable: $$O_{loc} = \frac{1}{n}\sum_{k=1}^{n} \mathbb{1}^{\otimes k-1} \otimes |0\rangle\langle 0| \otimes \mathbb{1}^{\otimes n-k} \quad \text{— (ii)}$$

Overhead Constrained PVQD



The circuit is composed of the gates arising from the Trotter step unitary $e^{-i\Delta t H}$ and gates in the variational ansatz state $|\psi(\theta)\rangle = U(\theta)|0\rangle$ we restrict to a 2-local Hamiltonian i.e. trotter expansion are given by two-qubit rotations $e^{-i\Delta t J_{ij} \sigma_i \otimes \sigma_j}$

$$e^{-i\Delta_t J_{ij} \sigma_i \otimes \sigma_j} \qquad \sigma_i, \sigma_j \in \{X, Y, Z\}$$

$J_{ij} \in \mathbb{R}$ coupling coefficients

Sampling overhead

$$\omega_{J_{ij}} = (1 + 2|\sin(2\Delta t J_{ij})|)^2$$

Δt small

$\Delta t J_{ij} << 1$

$\Rightarrow \omega_{J_{ij}} \simeq 1$

For 'L' such gates

$$\omega_{\Delta t} = \omega_J^L \qquad J_{ij} = J \;\forall i,j$$

While this scales exponentially in the number of gates, the base is small, and for a finite number of blocks, the overhead remains manageable.

OVERHEAD CONSTRAINED PVQD

$$\theta_t = \underset{\theta}{\operatorname{argmin}} \left[1 - |\langle \psi(\theta) | e^{-i\Delta t H} | \psi(\theta_{t-1}) \rangle|^2 \right]$$

PVQD evolves the parameter θ of a quantum circuit ansatz $|\psi(\theta)\rangle$ by maximizing the fidelity

The circuit corresponds of trotter gates (2-local Hamiltonians), such that the multiqubit gates appearing in the Trotter expansion are given by two-qubit rotations $e^{-i \Delta t J_{ij} \sigma_i \otimes \sigma_j}$ $\sigma_i, \sigma_j \in \{X, Y, Z\}$ and $J_{ij} \in \mathbb{R}$

The sampling overhead imposed by cutting a single instance of this gate with optimal decomposition is given by

$$\omega_{J_{ij}} = \left(1 + 2\left|\sin(2 \Delta t J_{ij})\right|\right)^2$$

two qubit rotations $R_{\sigma\sigma}(\theta) = \exp\left(-i \frac{\theta}{2} \sigma \otimes \sigma\right)$ for $\sigma \in \{X, Y, Z\}$ and $\theta \in [0, 2\pi]$

there is no advantage of using classical communication for a single gate instance, i.e

$$\gamma_{LOCC}(R_{\sigma\sigma}(\theta)) = \gamma_{\overrightarrow{LOCC}}(R_{\sigma\sigma}(\theta)) = \gamma_{LO}(R_{\sigma\sigma}(\theta)) = 1 + 2|\sin\theta|$$

Since one of our assumption is weak entanglement and small time steps; $\Delta t J_{ij} \ll 1$ and hence ω_J is close to 1.

$\omega_{\Delta t} = \omega_J^L$ for L such gate.