overhead for simulating a non-local channel with local channels by quasiprobability sampling.

Decompose non-local channels to linear combination of local channels and then make use of quasi-probability method for sampling.

channel robustness, cost of decomposition, resource reduction

weakness for the NISQ devices is that the number of qubits, the fidelities of gates and the connectivity are all limited. If we can somehow reduce the number of qubits or two-qubit gates required to obtain an output, it would widen the range of circuits that can be used for variational algorithms.

In the previous approach the overhead of decomposition is defined by the number of circuit runs that is required to achieve a desired accuracy of the output, and it scaled exponentially to the number of cuts performed. $O(9^k)$

forming a quasi probability decomposition of quantum channels

Quasiprobability distribution, defined by a set of complex numbers $\{q_i\}$ satisfying $\sum q_i = 1$

if a quantum channel ϕ can be decomposed as

$$\phi = \sum_i c_i \phi_i$$

where ϕ_i and c_i are channel and complex coeff.

then ϕ can be simulated by sampling ϕ_i with probability proportional to c_i.

Decomposition of non-local channels into local channels

Some notation: $|\rho\rangle\rangle$ density matrix ρ (also be seen as vector)
inner product $\langle\langle A|B\rangle\rangle = Tr(A^\dagger B)$

Channel Robustness of Non-locality

Single Qubit Rotation $R(n,\theta) = \exp\left[-i\theta\left(\sum_\alpha n_\alpha \sigma_\alpha\right)\right]$

Experimentally, the projective measurement is realized by rotating the axis by $R(n,\theta)$ and performing the projective measurement along z-axis.

The quantum channel $M(n)$ corresponding to projective measurement is a probabilistic map, when applied to a state $|\rho\rangle\rangle$, it returns a state

$$\frac{\Pi(\pm n)|\rho\rangle\rangle}{p_+}$$ with some prob p_+

where $\Pi(\pm n)$ is a projector to an eigenstate $\pm\sum n_\alpha\sigma_\alpha$ with eigenvalue $+1$

$\hat{\Pi}(n)$ ← probabilistic map

$$|\rho\rangle\rangle \xrightarrow{\hat{\Pi}(n)} \frac{\Pi(n)|\rho\rangle\rangle}{p_+}$$ (this with prob p_+)

$$|\rho\rangle\rangle \xrightarrow{\hat{\Pi}(n)} |0\rangle\rangle$$ This with probability p_-

This simply means ignore the case when the measurement result in -1

A new map.

$\tilde{\Pi}(n, c_+, c_-)$

$$|\rho\rangle\rangle \longrightarrow \frac{c_\pm \, \Pi(\pm n)\, |\rho\rangle\rangle}{p_\pm}$$

where $c_\pm \in \{0\} \cup \{e^{i\phi} \mid \phi \in [0, 2\pi]\}$

Expected value

$$E[|\sigma\rangle\rangle] = \sum_i p_i \, |\sigma_i\rangle\rangle$$

so this holds.

$$E\left[\tilde{\Pi}(n, c_+, c_-)\, |\rho\rangle\rangle\right] = c_+ \, \Pi(n)\, |\rho\rangle\rangle + c_- \, \Pi(n)\, |\rho\rangle\rangle$$